3. ZHELNOROVICH V.A., Models of Material Continua Possessing Internal Electromagnetic and Mechanical Moments. Izd-vo MGU, Moscow, 1980.
4. ZHELNOROVICH V.A., On the Newtonian equations for a fluid with internal magnetic and mechanical moments. Izv. Akad. Nauk SSSR, 6, 1974.
5. ZHELNOROVICH V.A., On the models of magnetizable and polarizable media with microstructure. Dokl. Akad. Nauk SSSR, 249, 2, 1979.
6. MAUGIN G.A., A phenomenological theory of ferroliquids, Int. J. Eng. Sci. 16, 12, 1978.
7. SUYAZOV V.M., On a non-symmetric model of a viscous electromagnetic fluid. PMTF, 2, 1970.
8. SHLIOMIS M.I., Effective viscosity of magnetic suspensions. Zhurn. Eksp. Teor. Fiz. 61, 6, 1971.
9. KASHEVSKII B.E., On the models of magnetic relaxation in ferrohydrodynamics. Magnit. Gidrodinamika, 5, 1978.
10. GOGOSOV V.V., NALETOVA V.A. and SHAPOSHNIKOVA G.A., Hydrodynamics of magnetizable media. Developments in Science and Technology. Mechanics of liquids and gases. 16, VINITI, Moscow, 1981.
11. ZHELNOROVICH V.A., Couette and Poiseuille flow of a viscous magnetizable fluid. Dokl. Akad. Nauk SSSR, 238, 2, 1978.
12. BATYAYEV I.M. and KOZLOVA A.N., Ferromagnetic resonance in a colloidal suspension of cobalt. Physical properties and hydrodynamics of disperse ferromagnetic materials. Izdpye UNTs Akad Nauk SSSR, Sverdlovsk, 1977.
13. TSEBERS A.O., Viscosity of a finely dispersed suspension of particles possessing cubic crystal symmetry in a magnetic field. Magnit. Gidrodinamika, 3, 1973.
14. SHLIOMIS M.I., Magnetic fluids. Uspeki Fiz. Nauk, 112, 3, 1974.
15. SEDOV L.I., Mathematical method of consturcting new models of continua. Uspekhi Mat. Nauk, 20, 5, 1965.
16. McTAQUE J.P., Magnetoviscosity of magnetic colloids, J. Chem. Phys. 51, 1, 1969.
17. MOZGOVOI E.N., BLUM E.YA. and TSEBERS A.O., Flow of a ferromagnetic fluid in a magnetic field. Magnitn. Gidrodinamika, 1, 1973.
18. MAIOROV M.M., Measuring the viscosity of a ferrofluid in a magnetic field. Magnitn. Gidrodinamika, 4, 1980.

Translated by L.K.

PMM U.S.S.R.,Vol.51,No.4,pp.548-551,1987
Printed in Great Britain

0021-8928/87 $10.00+0.00

ON THE PAPER BY V.A. ZHELNOROVICH ENTITLED "ON MATHEMATICAL MODELS OF MAGNETIC FLUIDS"*

V.V. GOGOSOV

The paper by V.V. Zhelnorovich /1/ consists of two parts. In the first part equations are given for describing magnetic fluids (MF), which are not basically different from the well-known euqations /2-4/. The system is linearized and a solution is sought for the propagation of monochromatic waves. In the second part of the paper (Sect.5) an attempt is made to reply to a criticism which appeared in the review /4/ and in /3/, concerning the paper by Zhelnorovich /5/. Further, another model of MF is described in Sect.5, different from that given in Sect.1 of /1/, and is used to solve the Couette and Poiseuille problems. In doing this he not only repeats the errors already discussed in /3, 4/, but he also makes further errors, which will be discussed in the present paper.

The interest in describing the behaviour of magnetizable fluid media in nagnetic fields is primarily connected with producing MF and practical applications of MF, which are colloidal solutions of fine ferromagnetic particles.

It would seem that the simplest way of describing the behaviour of MF would be to use the normal equations of hydrodynamics with an additional force $\mathbf{M}\nabla\mathbf{H}$ acting on the fluid from the direction of the magnetic field $\mathbf{H}$. The magnetization $\mathbf{M}$ can, in most cases, be assumed to be parallel to the magnetic field $\mathbf{M} = \chi\mathbf{H}$. Precisely such a model was proposed in /6/. It satisfactorily describes many phenomena and is widely used in practice.

**Prikl.Matem.Mekhan.*,51,4,700-703,1987

However, experiments have brought to light many phenomena which cannot be described within the framework of this model. Such phenomena include the rotation of MF under the action of a uniform rotating magnetic field, the increase in effective viscosity of MF by 20—25% when a uniform magnetic field is imposed on it, etc. The force $M\nabla H$ in these experiments is equal to zero, and the model of /6/ implies that the magnetic field should not affect the motion of MF, and this contradicts the experimental data given in e.g./7-12/.

One of the factors causing the motion of MF in a uniform magnetic field is the rotation of the dispersed ferromagnetic particles under the action of a rotating, although uniform, magnetic field, and of the moments of random Brownian forces. In this case the hydrodynamic aspects influence the magnetization of the medium, so that the magnetization is not determined by the instantaneous value of the magnetic field $M \neq \chi H$. This physical mechanism of the effect of the field was apparently first mentioned in /8, 13/. A mathematical model of MF describing this physical mechanism was given in /14, 15/.

It should be noted that the above models are widely used, and their simplest versions describe analytically and with an accuracy to within the order of the parameter, the influence of a one-dimensional magnetic field on the hydrodynamic properties of MF observed experimentally /7-12/.

Another mechanism of the effect of a magnetic field on the medium has been known for a long time, namely, the gyromagnetic effect, which consists, in particular, of the appearance of the precession term in the equation for the magnetization. The equations taking into account the gyromagnetic effect have been given in a number of papers, e.g. /16, 17/.

The paper by Zhelnorovich /5/ employs the equations utilizing only the gyromagnetic effect to solve the problem of Couette and Poiseuille flows. He asserts there that the equations given in this paper "can be used e.g. to describe ferromagnetic fluids".

It was this specific assertion that attracted our attention. The point is, that up to the present time Zhelnorovich has published over ten papers (see e.g. /18-26/) dealing with the same problem: the problem of describing magnetizable fluids. The equations derived or quoted by him in various papers differ from each other. No reason is given for this discrepancy, and not a single paper mentions precisely which pysical processes and which specific medium are described by one model or another.

On the other hand, experiments involving MF have been carried out many times /7-12/. However, Zhelnorovich has made no comparison between the theoretical and experimental results, either in /5/, or in any other paper.

Such a comparison was made by us in /4/ and by Kashevskii in /3/. The characteristic parameters in the estimates corresponded to the experimental data. The conclusion reached by us was that the increase in the viscosity calculated using the formulas in /5/ were 3-4 orders of magnitude smaller than those observed experimentally.

Further, in /4/ it was shown that by taking into account the rotation of ferromagnetic particles (which was neglected in the paper by Zhelnorovich) one can describe the results of the experiments in /7-12/. Therefore, the final conclusion reached in /4/ was, that the Zhelnorovich equations in /5/ were unsuitable for describing MF flows.

The reply by Zhelnorovich to the criticisms is published in this number of the journal /1/. His reply again contains a new series of equations for describing the same "magnetic", "ferromagnetic" and "magnetizable" fluids, which again differ from the equations he gave earlier. Moreover, even the equations he gives in the first and second section of the paper differ substantially from each other (!). Again, no explanations are given.

There is not the slightest attempt to compare the results obtained with the experimental data from /7-12/, as if the latter did not exist. Lengthy discussions are carried out concerning the possibility of the existence of "various" MF, in particular, of fluids for which the increase in viscosity in a magnetic field caused by the rotation of ferromagnetic particles would be even smaller than the gyromagnetic effect. The only response to this can be: in this case neither the rotation of the particles nor the gyromagnetic effect need be considered, at least in the Couette and Poiseuille problems with which the paper /5/ is exclusively concerned.

In Sect.5 of /1/ three paragraphs contain a discussion of the paper by Shliomis /15/ in which the Couette problem was solved using a model which takes into account the rotation of the particles, and the result obtained in in good agreement with experimental data: the ratio of the change in viscosity in a magnetic field $\Delta\eta$ to the viscosity without the field η is of the order of 20—30%. In the first paragraph of /1/ Zhelnorovich says that this change is "small" and can be neglected, and the MF in which such an increase in viscosity occurs "can be described with sufficient accuracy by the equations of /5, 6/".

This statement is erroneous. How can one describe experiments in MF flows in which the effective viscosity increase by 20—25% using the theory of Zhelnorovich /5/ in which the relative change in viscosity is $\Delta\eta/\eta \sim 5\cdot 10^{-3} - 5\cdot 10^{-4}$?

In the next two paragraphs the author states the well-known result from /27/, that the time derivative of the internal angular momentum $(dI\Omega/dt)$ can be neglected in the equation of internal angular momentum. Here the algebraic equation that remains can be used to obtain an expression for $I\Omega$ and one can substitute it into the other equations. This leads Zhelnorovich to the conclusion that "the use of the moment $I\Omega$ in existing theories /14, 15/ is without any foundation whatsoever".

This assertion is also erroneous. We could attempt to state with equal success that the equation of state $p=\rho RT$ can be used to eliminate the pressure from the other equations of hydrodynamics, and conclude that introducing the pressure p "is without any foundation whatsoever" (?!).

Let us now turn to the simplest Couette and Poiseuille problems solved by Zhelnorovich in Sect.6 and 7 of /1/.

It would seem that in response to our criticism of /5/, Zhelnorovich solves the Couette and Poiseuille problems in /1/ using the system of equations given in Sect.5, in which the influence of the magnetic field on the rotation of ferromagnetic particles is taken into account, as was done repeatedly by other authors, e.g. in /2, 4, 17/. As we already said, the system differs not only from the one given in Sect.1, but also from the corresponding system of /5/ in which the same Couette problem was solved. The difference consists of the fact that the equation for the magnetization contains no terms of a gyromagnetic nature $g[\boldsymbol{M}, \boldsymbol{H}]$ whatsoever.

This is precisely the reason why, when solving the Couette problem, the terms connected with the gyromagnetic effect make no contribution whatsoever in the equations of Sect.5 of /1/, to the magnitude of the increase in effective viscosity, while in /5/, which dealt with a solution of the same Couette problem, the total increase in viscosity was determined by the gyromagnetic effect. Again no explanations are offered.

The increase in the viscosity in the Couette problem in /1/ is determined only by the mechanical rotation of the particles. It is therefore not surprising that the expression given by Zhelnorovich for the effecitve viscosity in Couette flow is a special case of the formula obtained by Shliomis in the course of solving the same problem in /15/ as far back as 1971.

In concluding the paper /1/ Zhelnorovich writes that estimating the terms connected with the gyromagnetic effect "in the solutions obtained above shows that *in the case of real fluids the terms are, generally speaking, small, and their influence on the flows studied here can practically be neglected*" (my italics). But it is precisely that statement that appeared in /4/ and /2, 3/. What purpose does it serve to solve once again the elementary Couette and Poiseuille problems? Is it to confirm them once again? What is the purpose of carrying on a lengthy discussion in Sect.5 about the existence of "various" magnetic fluids for which the gyromagnetic effects are important, if they are insignificant in the case of "real" (!) fluids. What, in fact, is the paper /1/ about?.

REFERENCES

1. ZHELNOROVICH V.A., On mathematical models of magnetic fluids. PMM, 51, 4, 1987.
2. TSEBERS A.O., Common rotational diffusion of the magnetic moment and rigid matrix of a single domain ferroparticle. Magnitn. Gidrodinamika, 3, 1975.
3. KASHEVSKII B.E., On the models of magnetic relaxation in ferrohydrodynamics. Magnitn. Gidrodinamika, 4, 1978.
4. GOGOSOV V.V., NALETOVA V.A. and SHAPOSHNIKOVA G.A., Hydrodynamics of magnetizable fluids. Developments in Science and Technology. Ser.Mechanics of liquid and gases. VINITI, Moscow, 16, 1981.
5. ZHELNOROVICH V.A., Couette and Poiseuille flow of a viscous magnetizable fluid. Dokl. Akad. Nauk SSSR, 238, 2, 1978.
6. NEURINGER J.L. and ROZENSWEIG R.E., Ferrohydrodynamics, Phys. Fluids. 7, 12, 1964.
7. McTAQUE J.P., Magnetoviscosity of magnetic colloids, J. Chem. Phys. 51, 1, 1969.
8. MOSCOVITZ R. and ROZENSWEIG R.E., Non-mechanical torque-driven flow of a ferromagnetic fluid by an electromagnetic field. Appl. Phys. Lett. 11, 10, 1967.
9. MAILFERT R. and MARTINET A., Flow regimes for a magnetic suspension under a rotating magnetic field, J. Phys. 34, 2-3, 1973.
10. KAGAN I.YA., RYKOV V.G. and YANTOVSKII E.I., On a flow of a dielectric ferromagnetic suspension in a rotating magnetic field. Magnitn. Gridrodinamika, 2, 1973.
11. MAIOROV M.M., Measuring the viscosity of a ferrofluid in a magnetic field. Magnitn. Gidrodinamika, 4, 1980.
12. MOZGOVOI E.N., BLUM E.YA. and TSEBERS A.O., Flow of a ferromagnetic fluid in a magnetic field. Magnitn. Gidrodinamika, 1, 1973.
13. HALL W.F. and BUSENBERG S.N., Viscosity of magnetic suspensions, J. Chem. Phys. 51, 1, 1969.
14. SUYAZOV V.M., On a non-symmetric model of a viscous electromagnetic fluid. PMTF, 2, 1970.

15. SHLIOMIS M.I., Effective viscosity of magnetic suspensions. Zh. Eksperim. i Teoret. Fiziki, 61, 6, 1971.
16. LANDAU L.D. and LIFSHITZ E.M., Electrodynamics of Continua. Nauka, Moscow, 1982.
17. SHLIOMIS M.I., On the equations of motion of a fluid possessing gyromagnetic properties. Zh. Eksp. Teor. Fiz. 53, 3, 1967.
18. ZHELNOROVICH V.A., On the energy-momentum tensor of electromagnetic field in connection with a model of magnetizable and polarizable fluid with internal angular momentum. Dokl. Akad. Nauk SSSR, 217, 5, 1974.
19. ZHELNOROVICH V.A., On Newtonian motions for fluids with internal angular and mechanical moments. Izv. Akad. Nauk SSSR, MZhG, 6, 1974.
20. ZHELNOROVICH V.A., Magnetizable and polarizable continua with internal angular momentum in Newtonian mechanics. Magnitn. Gidrodinamika, 2, 1977.
21. ZHELNOROVICH V.A., Integrals of Newtonian equations and non-stationary one-dimensional exact solutions for the models of magnetizable fluids with internal angular momentum. Magnitn. Gidrodinamika, 1, 1978.
22. ZHELNOROVICH V.A., One-dimensional waves in magnetizable fluids. Proceeding of the All-Union Seminar on Problems of Magnetizable Fluids. Ivanovo, 1978.
23. ZHELNOROVICH V.A., On the models of magnetizable and polarizable media with a microstructure. Dokl. Akad. Nauk SSSR, 249, 2, 1979.
24. ZHELNOROVICH V.A., One-dimensional non-stationary waves in magnetizable fluids with internal angular momentum. Magnitn. Gidrodinamika, 1, 1979.
25. ZHELNOROVICH V.A., Models of Material Continua with Internal Angular and Mechanical Moments Izd-vo MGU, Moscow, 1980.
26. ZHELNOROVICH V.A. Magnetizable and polarizable media with microstructure. Macroscopic theories of matter and fields: A thermodynamic approach, Ed. by L.I. Sedov. Moscow, MIR, 1983.
27. SHLIOMIS M.I., Magnetic fluids. Uspekhi Fiz. Nauk, 112, 3, 1974.

Translated by L.K.